Question Paper

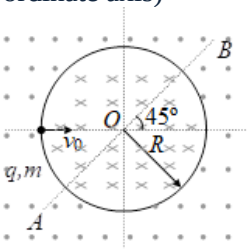
Physics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A particle having mass 0.5kg and charge 0.2C is projected with a speed $V_0 = 10$ m/s from the periphery of a circle of radius R = 5m towards the centre as shown in the figure such that region inside the circle carries a uniform magnetic field of strength $B_0 = 5T$ into the plane and in the outside region has a uniform magnetic field of the same strength coming out of the plane then, (Assume the center O to be the origin of the co-ordinate axis)



A. the motion of the charged particle is periodic
B. the particle crosses line AB after nearly 5.1 s of its start from the periphery
C. the force on the charged particle due to uniform magnetic field is 20 N
D. the co-ordinates of the center of the circle at t = 2s is (5, 5)

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Two radioactive material A_1 and A_2 have decay constants of $10\lambda_0$ and λ_0. If initially they have same number of nuclei, the ratio of number of their undecayed nuclei will be (1/e) after a time

A. $\frac{1}{\lambda_0}$
B. $\frac{1}{9\lambda_0}$
C. $\frac{1}{10\lambda_0}$
D. 1

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

An object of mass m is at rest in equilibrium at the origin. At t = 0 a force $\vec{F}(t)$ is applied which has the components
$F_x(y) = K_1 + K_2 y$
$F_y(t) = K_3 t$
Where K_1, K_2, K_3 are constants. Then

A. X-component of its velocity at any time â€˜tâ€™ is $v_X = \frac{K_1 t}{m} + \frac{K_2 K_3}{24m^2} t^4$
B. X-component of acceleration at any time t is $(\frac{K_1}{m} + \frac{K_2 K_3}{6m^2} t^3)$
C. Y-coordinate of object at any time â€˜tâ€™ is $\frac{K_3 t^2}{3m}$
D. none of these

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Consider a hypothetical hydrogen like atom in which electrostatic potential between electron and nucleus is given by $V = \frac{k}{r^3}$ where k is a constant. If n be the principle quantum number of the orbit, v = speed of the electron, r = radius of the orbit of electron. Assuming Bohr's theory valid for this atom,

A. $v \propto \frac{1}{n}$
B. $v \propto n^{-3}$
C. $r \propto \frac{1}{n^2}$
D. $r \propto n^0$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A monatomic gas undergoes a thermodynamic process such that $T \propto V^{-\frac{2}{7}}$
(T = temperature and V = volume of the gas)
As heat is supplied to the gas, mark incorrect options

A. rms velocity of gas molecules increases
B. rms velocity of gas molecules decreases
C. work done on the gas is positive
D. density of the gas increases

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In a screw gauge, there are 100 divisions in circular scale and each main scale division is of 1 mm. When there is no gap between the jaws, 97th divisions coincides with the main scale zero and zero of main scale is not visible. While measuring the diameter of a ball, the circular scale is between 3 mm mark and 4 mm mark such that the 76th division of circular scale coincides with the reference line. Select the incorrect alternative

A. the least count of the micrometer is 0.01 cm
B. the zero error is -0.04 mm
C. the diameter of the ball is 3.79 cm
D. the main scale reading is 4 mm

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The equation of standing wave in a string of mass 100 g length 1m and fixed at both ends is given by $y = \frac{\sqrt{2}}{10} \sin\frac{\pi x}{10} \cos\frac{\pi t}{5}$, which of the following is/are correct? (x and y are in cm and t in sec)

A. number of nodes is 10
B. tension in the string is $4 \times 10^5 N$
C. two points having amplitudes 0.1 cm each are separated by 5 cm
D. potential energy of antinode is zero

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If the two slits of double slit experiment were moved symmetrically apart with small relative velocity v and the distance between screen and mid point of slits is fixed and equal to D. Consider a point P on the screen at a distance x from central maxima then -

Rate of change of number of fringes between central maxima and point P changes with respect to time

A. is $\frac{xv}{\lambda D}$

B. number of fringes contained between central maxima and point P increases with time

C. fringe width decreases as time passes

D. fringe width increases as time passes

Physics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

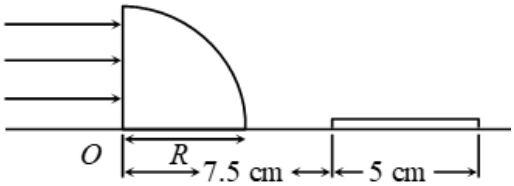
The light of radiation 300 nm falls on a photocell operating in the saturation mode. The spectral sensitivity is 4.8 mA/Watt. The yield of photoelectrons (i.e. number of electrons produced per photon) is K, Find 100K and roundoff to nearest integer.

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

A glass $(\mu = 2)$ prism is in the shape of a quarter cylinder of length L and radius $R = 5$ cm , lying on a horizontal table. A uniform horizontal light beam fall on its vertical plane surfaces, as shown in figure. If a plane paper of width 5 cm and length L is placed at a distance 7.5 cm from O, then the fraction of light (in percentage) falling on the prism that will fall come out of prism is _____

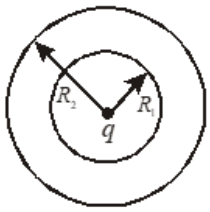


Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

The region between two concentric spheres of radii R_1 and $R_2 (R_1 < R_2)$ has volume charge density $\rho = b/r$, where b is constant and r is the radial distance. A point charge $q = 16\mu$C is placed at the origin, $r = 0$. Find the value of b (in SI units) for which the electric field in the region between spheres is constant. (take $R_2 = 2R_1 = \frac{4}{\sqrt{\pi}}$ mm)



Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

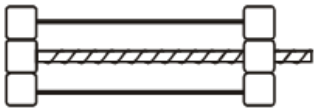
The minimum colatitude which can directly receive a signal from a geostationary satellite can be written as $\sin^{-1}\left(\frac{1}{\alpha}\right)$ find $\alpha =$? (closest integer)

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

A steel bolt of cross-sectional area $A_b = 5 \times 10^{-5}\ m^2$ is passed through a cylindrical tube made of aluminium. Cross-sectional area of the tube material is $A_t = 10^{-4}\ m^2$ and its length is $\ell = 50$ cm. The bolt is just taut so that there is no stress in the bolt. And temperature of the assembly is increases through $\Delta\theta = 10^{\circ}C$. Given, coefficient of linear thermal expansion of steel, $\alpha_b = 10^{-5/^{\circ}C.}$



Young's modulus of steel $Y_b = 2 \times 10^{11}\ N - m^{-2}$.
Young's modulus of Al, $Y_t = 10^{11}\ N - m^{-2}$, coefficient of linear thermal expansion of Al is $\alpha_t = 2 \times 10^{-5}/^{\circ}C$. The compressive strain in tube is ____$\times 10^{-5}$.

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

A particle is projected upwards with a velocity of 100 m/sec at an angle of 60° with the vertical. Find the time when the particle will move perpendicular to its initial direction, taking $g = 10\ m/sec^2$.

Physics Paragraph Type (Maximum Marks: 12)

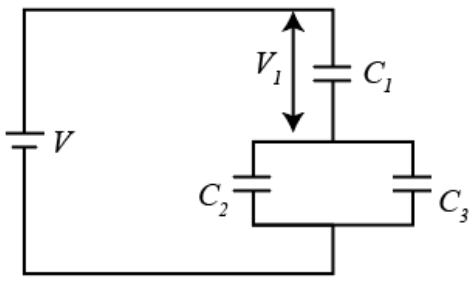
Question No. 1

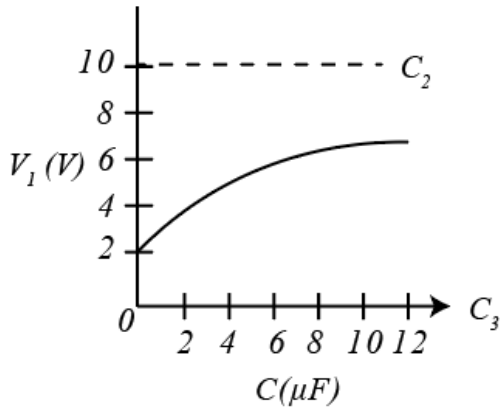
Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

Capacitor C_3 in the circuit is a variable capacitor (its capacitance can be varied). Graph is plotted between potential difference V_1 (across capacitor C_1) versus C_3. Electric potential V_1 approaches on asymtote of 10 V as $C_3 \to \infty$.





Question

The electric potential V across the battery is equal to:

A. 10 V
B. 12 V
C. 16 V
D. 20 V

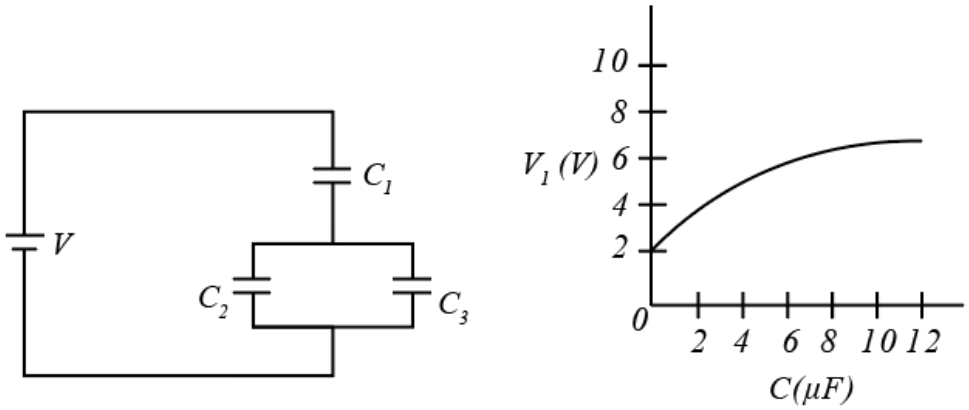
Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

Capacitor C_3 in the circuit is a variable capacitor (its capacitance can be varied). Graph is plotted between potential difference V_1 (across capacitor C_1) versus C_3. Electric potential V_1 approaches on asymtote of 10 V as $C_3 \rightarrow \infty$.



Question

C_1/C_2 has value:

A. 4
B. 1/4
C. 2
D. 1/2

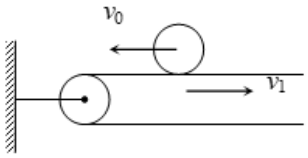
Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

A sphere of radius r and mass m has a linear velocity v_0, directed to the left and no angular velocity as it is placed on a belt at $t = 0$. The belt is moving to the right with a constant velocity v_1. If after sliding on the belt the sphere has no linear velocity of its centre of mass with respect to ground as it starts pure rolling on the belt without sliding at $t = t_1$, and coefficient of friction between the belt and the sphere is μ_k.



Question

The required value of v_0

A. $\frac{3}{5} v_1$
B. $\frac{2}{5} v_1$
C. v_1
D. $\frac{7}{5} v_1$

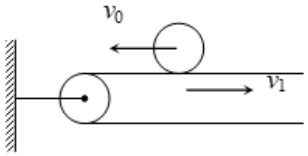
Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

A sphere of radius r and mass m has a linear velocity v_0, directed to the left and no angular velocity as it is placed on a belt at $t = 0$. The belt is moving to the right with a constant velocity v_1. If after sliding on the belt the sphere has no linear velocity of its centre of mass with respect to ground as it starts pure rolling on the belt without sliding at $t = t_1$, and coefficient of friction between the belt and the sphere is μ_k.



Question

The distance moved by sphere relative to belt in time t_1

A. $\frac{2}{25}\frac{v_1^2}{\mu g}$

B. $\frac{9}{50}\frac{v_1^2}{\mu g}$

C. $\frac{12}{25}\frac{v_1^2}{\mu g}$

D. $\frac{8v_1^2}{25\mu g}$

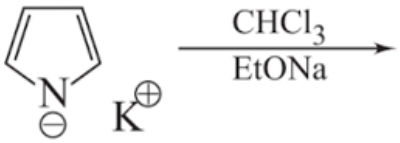
Chemistry Multiple Correct (Maximum Marks: 32)

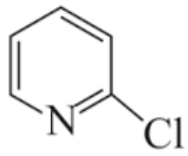
Question No. 1

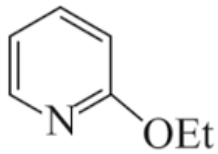
One or More Options Correct Type

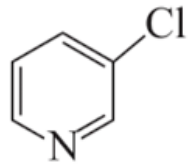
The question has multiple options out of which ONE or MORE is/are correct.

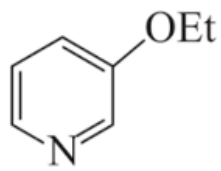
The major product formed in the following reaction is



A. 

B. 

C. 

D. 

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Acid-catalysed reaction of D-glucose with benzaldehyde produces the 4,6-O-benzylidene derivative. Reduction with $NaBH4$, followed by excess $HIO4$ cleavage and acid hydrolysis yields a $C_4H_8O_4$ tetrose and benzaldehyde. What is the configuration of this tetrose?

A. 2 S, 3 S
B. $2R, 3S$
C. 2R, 3R
D. 2 S, 3R

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Zinc oxide is normally white but turns yellow on heating and becomes white again on cooling, because of (select incorrect option)

A. d-d transition spectra as well as a crystal defect
B. the two-dimensional network structure of ZnO
C. its high transition temperature (T_c)
D. various types of lattice defects because of which the oxygen ion is lost during heating

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Match the following :

	Column-I		Column- II
(a)	$t_{1/2} \propto a^{-1}$ (a = initial concentration of reactants)	(p)	$E_f < E_b$ (E_f and E_b are activation energies of forward and backward reaction)
(b)	Rate constant (k) has unit = $Lmol^{-1}\ s^{-1}$	(q)	Order of reaction is 0.
(c)	Equilibrium constant (K) has infinitely large value.	(r)	Order of reaction is 2.
(d)	$t_{1/2}$ decreases linearly with time	(s)	$[A] = \frac{[A]_0}{1+kt[A]_0}$ $[A]_0$ = initial concentration , [A] = instantaneous concentration , k = rate constant.

A. $\begin{array}{|c|c|c|c|}\hline a & b & c & d \\ \hline r,s & r,s & p & q \\ \hline \end{array}$

B. $\begin{array}{|c|c|c|c|}\hline a & b & c & d \\ \hline r,s & r & q & p \\ \hline \end{array}$

C. $\begin{array}{|c|c|c|c|}\hline a & b & c & d \\ \hline r & rs & q & p \\ \hline \end{array}$

D. $\begin{array}{|c|c|c|c|}\hline a & b & c & d \\ \hline r,s & s & p & q \\ \hline \end{array}$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Among the following which reactions are possible

A. $F_2 + H_2O \longrightarrow HF + O_2$
B. $Cl_2 + H_2O \longrightarrow HCl + HClO$
C. $Br_2 + H_2O \longrightarrow HBr + HBrO$
D. $I_2 + H_2O \longrightarrow Hl + HIO$

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

SRP of Ag|AgCl electrode is $0.2\ V$ and that of a Ag electrode is $0.79\ V$. The max. amount of $AgCl$ that can dissolve in 10^6 L of a $0.1M AgNO_3$ solution is

A. 0.5 mmol
B. 1.0 mmol
C. 2.0 mmol
D. 2.5 mmol

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Which of the given statements is true for the following reaction?
$[Cu(H_2O)_4]^{2+} + 4NH_3 \rightleftharpoons [Cu(NH_3)_4]^{2+} + 4H_2O$

A. It is a ligand-substitution reaction.
B. NH_3 is a relatively strong-field ligand while H_2O is a weak-field ligand.
C. During the reaction, there is a change in colour from light blue to dark blue.
D. $[Cu(NH_3)_4]^{2+}$ has a tetrahedral structure, and is paramagnetic.

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

How many of the following are arranged in the increasing order of the property indicated against them?

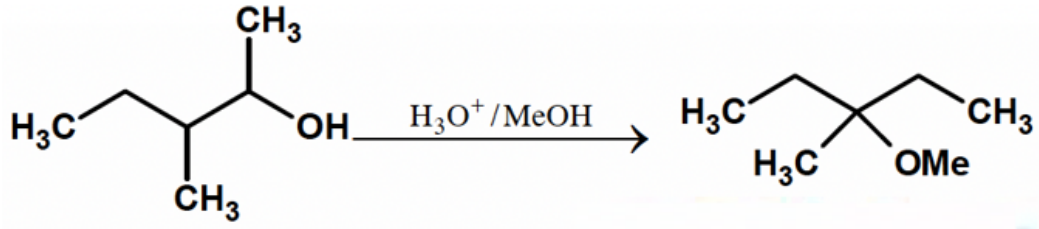
A. $CH_3OH < CH_3COOH^- CH_3OH_2^+$: Acidic character
B. $CH_3COONa < PhONa < EtONa$: Basic character
C. $: CBr_2 <: CCl_2 <: CF_2$: Stability
D. $MeSH < MeOH < EtOH < PhOH$: Boiling point

Chemistry Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.



No. of transition states in the above conversion = X
No. of intermediates which are more stable than the intermediate formed in the third step during course of reaction = y
Report your answer as (X + Y + 1)/2 = ?

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Calculate the number of moles of AgI, which may be dissolved in 1.0 L of 1.0 M CN^- solution. K_{SP} for AgI and K_C for $Ag(CN)_2^-$ are $1.2 \times 10^{-17}\ M^2$ and $7.1 \times 10^{19} M^{-2}$ respectively (roundoff to one decimal place).

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

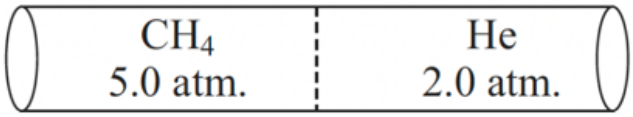
Consider the Equilibrium:
$NH_2COONH_4(s) \rightleftharpoons N_2(g) + 3H_2(g) + CO(g) + \frac{1}{2} O_2(g)$
The value of K_p (in suitable atm units) is $3^x 2^y$ and the total pressure at equilibrium is 11 atm. What is the value of $x + y$?

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

A long cylindrical glass tube, equipped with a porous disc at the centre, contain methane gas at 5.0 atmosphere on one side and He gas at 2.0 atmosphere on the other side of the disc as shown in the diagram below:
Disc is permeable to both gases and rate of diffusion is directly proportional to the gas pressure and inversely proportional to square root of molar masses as:



$-\frac{dp}{dt} = k\frac{P}{\sqrt{M}}$ where, k is a constant.
If k for the diffusion of methane gas is 2.5×10^{-2} second $^{-1}$, determine time after which pressure of methane chamber will drop to 4.0 atmospheres (nearest integer).

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

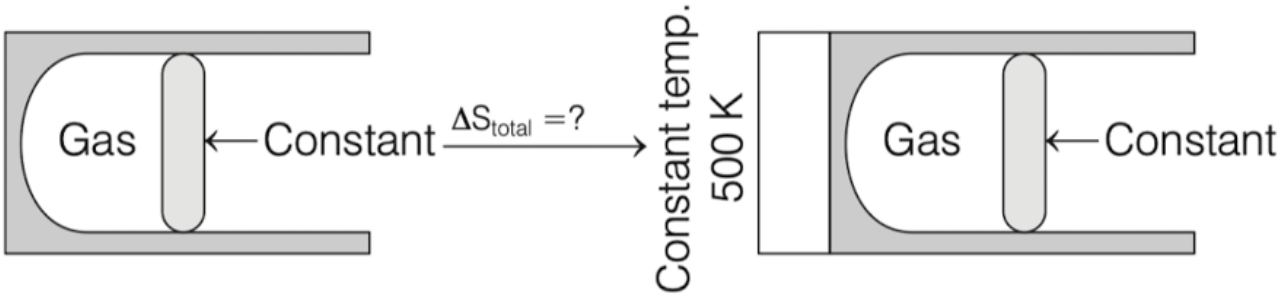
A galvanic cell consist of a Zn electrode in a $0.1M Zn^{2+}$ solution and a second half-cell with a Pt electrode in 0.1M solution of HIO_3. Calculate the cell potential if K_a of HIO_3 is 0.20. [Given :
$E^\circ(Zn^{2+}/Zn) = -0.76$ V]

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

3 moles of an ideal gas ($C_V, m = \frac{3}{2} R$) are initially in an insulated piston - fitted cylinder at a constant pressure. The volume of the gas is initially $40\ \mathrm{L}$. The cylinder is then placed in contact with a constant temperature bath at $500\ \mathrm{K}$. Bringing the bath in contact with the cylinder causes $6.236\ \mathrm{kJ}$ of heat to transfer into the cylinder and causes the gas to expand. The value of ΔS total for this process is _____ in Joules (nearest integer)



Chemistry Paragraph Type (Maximum Marks: 12)

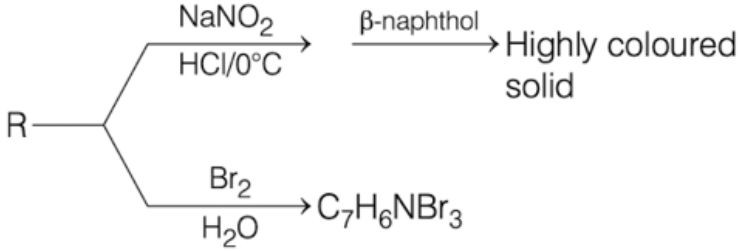
Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

A solid organic ' P ' of formula $C_{15}H_{15}ON$, was found to be insoluble in water, dil. HCl or dil. NaOH in cold. After prolonged heating of ' A ' with aqueous NaOH, a liquid ' R ' was found to be floating on the surface of alkaline mixture. ' R ' did not solidified on cooling to room temperature. It was steam distilled and separated. Also, acidification of alkaline mixture with hydrochloric acid caused precipitation of a white solid ' S ' $(C_8H_8O_2)$. Some additional informations are as follows :

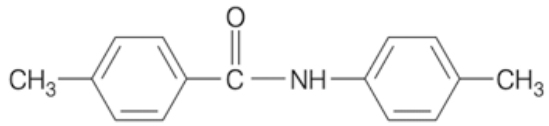


Son treatment with $Br_2/FeBr_3$ in CCl_4 produced a single isomer $C_8H_7O_2Br$ while heating ' S ' with soda lime gave toluene.

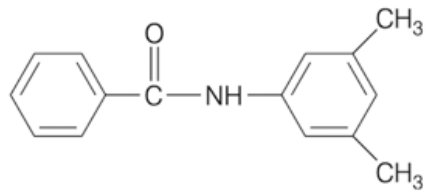
Question

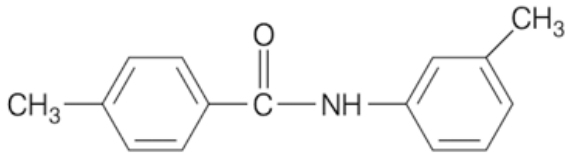
The most likely structural formula of ' P ' is

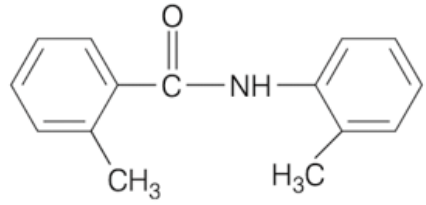
A.



B.



C. 

D. 

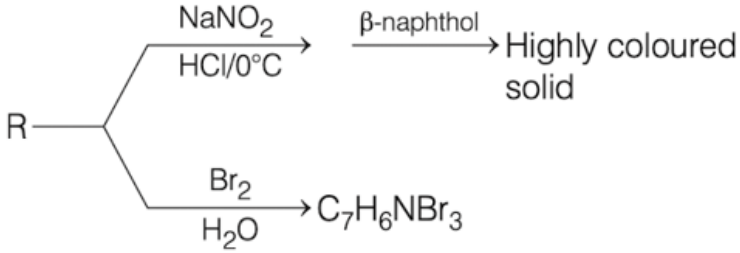
Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

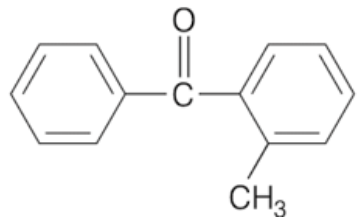
A solid organic ' P ' of formula $C_{15}H_{15}ON$, was found to be insoluble in water, dil. HCl or dil. NaOH in cold. After prolonged heating of ' A ' with aqueous NaOH, a liquid ' R ' was found to be floating on the surface of alkaline mixture. ' R ' did not solidified on cooling to room temperature. It was steam distilled and separated. Also, acidification of alkaline mixture with hydrochloric acid caused precipitation of a white solid ' S ' $(C_8H_8O_2)$. Some additional informations are as follows :

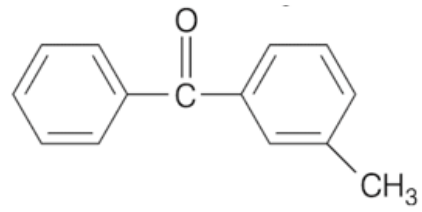


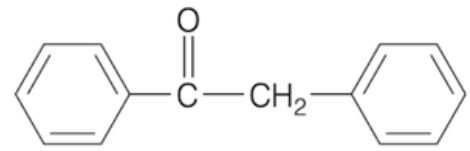
Son treatment with $Br_2/FeBr_3$ in CCl_4 produced a single isomer $C_8H_7O_2Br$ while heating ' S ' with soda lime gave toluene.

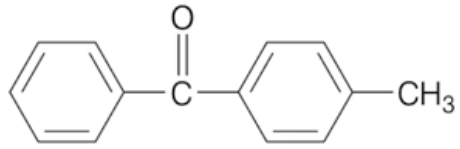
Question

It S is treated with $SOCl_2$ followed by addition of benzene and some $AlCl_3$ would produce

A. 

B. 

C. 

D. 

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

Athletes sometimes disturb the buffering of their blood through a process called hyperventilation. In the excitement and anxiety of a contest, they breathe more rapidly and deeply they need to. When hyperventilation occurs, a person expels more CO_2 than necessary, upsetting the carbonic acid equilibrium. According to Le-Chatelier's principle (LCP) as the CO_2 is exhaled, more H_2CO_3 decomposes to replace the CO_2 in the reaction

$H_2CO_3 \rightleftharpoons H_2O + CO_2$

As H_2CO_3 is used, the equilibrium between carbonic acid and hydrogen carbonate gets disturbed and HCO_3^- gets consumed in replacing the H_2CO_3.

$HCO_3^- + H_3O^+ \rightleftharpoons H_2CO_3 + H_2O;$

Eventually, the HCO_3^- concentration drops to the point at which it is insufficient to maintain the blood pH at a safe level. As HCO_3^- also consumes H_3O^+, the blood pH is raised (becomes more basic). Once response of the body to this condition is achieved, there is constriction of the cerebral blood vessels. As the blood flow to the brain is reduced, the individual becomes dizzy and can lapse into unconsciousness. At that point the body's reflex mechanism usually restore normal breathing and the blood pH returns to normal.

Question

To maintain a pH of 7.4 for blood at normal condition, which is 2M in H_2CO_3 (at equilibrium), what volume of 5M $NaHCO_3$ solution is required to mix with 10 mL of blood? [Given that pK_a for H_2CO_3 in blood is 6.002 and $\log 2 = 0.3010$]

A. 100 mL
B. 150 mL
C. 50 mL
D. 25 mL

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

Athletes sometimes disturb the buffering of their blood through a process called hyperventilation. In the excitement and anxiety of a contest, they breathe more rapidly and deeply they need to. When hyperventilation occurs, a person expels more CO_2 than necessary, upsetting the carbonic acid equilibrium. According to Le-Chatelier's principle (LCP) as the CO_2 is exhaled, more H_2CO_3 decomposes to replace the CO_2 in the reaction

$H_2CO_3 \rightleftharpoons H_2O + CO_2$

As H_2CO_3 is used, the equilibrium between carbonic acid and hydrogen carbonate gets disturbed and HCO_3^- gets consumed in replacing the H_2CO_3.

$HCO_3^- + H_3O^+ \rightleftharpoons H_2CO_3 + H_2O;$

Eventually, the HCO_3^- concentration drops to the point at which it is insufficient to maintain the blood pH at a safe level. As HCO_3^- also consumes H_3O^+, the blood pH is raised (becomes more basic). Once response of the body to this condition is achieved, there is constriction of the cerebral blood vessels. As the blood flow to the brain is reduced, the individual becomes dizzy and can lapse into unconsciousness. At that point the body's reflex mechanism usually restore normal breathing and the blood pH returns to normal.

Question

The administration of a medicine containing $H_2PO_4^-$ in significant amount will

A. increase the exhaling of CO_2
B. decrease the exhaling of CO_2
C. doesn't affect the liberation of CO_2
D. none of these

Mathematics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let $f: R \to R, g: R \to R$ be two continuous function satisfying equation $f(x) + f(x+1) = x + \{x\}; g(x) + g(2-x) = 2$, then

A. $\int_{-1}^{1} f(x+1)dx = 2$
B. $\int_{-1}^{1} f(x+1)dx = 1$
C. $\int_{1/2}^{3/2} g(g(x))dx = 2$
D. $\int_{1/2}^{3/2} g(g(x))dx = 1$

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

$x + y = 2$ and $x - y = 2$ are tangents on a parabola at $(1, 1)$ and $(4, 2)$ respectively. Which of the followings is/are correct.

A. Equation of directrix is $x + 3y = 2$
B. Equation of axis is $3x - y = 5$
C. Focus of the parabola is at $(\frac{8}{5}, \frac{6}{5})$
D. Vertex of the parabola is at $(\frac{33}{20}, \frac{13}{20})$

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If $\alpha = \tan^{-1}(1) + \frac{1}{2}\tan^{-1}(2) + \frac{1}{3}\tan^{-1}(3), \beta = \tan^{-1}(1) + 2\tan^{-1}(\frac{1}{2}) + 3\tan^{-1}(\frac{1}{3})$ and $|\alpha - \beta| = \frac{p\pi}{q} + \frac{r}{s}\cot^{-1}(3)$, where $p, q, r, s \in N$ and are in their lowest form, then which of the following is/are INCORRECT?

A. $p - r = 0$
B. $q = 4s$
C. $p + q + r + s = 42$
D. $pr = 1 + q$

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The equations of four circles are $(x \pm a)^2 + (y \pm a)^2 = a^2$. The radius of a circle touching all the four circles is

A. $(\sqrt{2} - 1)a$
B. $2\sqrt{2}a$
C. $(\sqrt{2} + 1)a$
D. $(2 + \sqrt{2})a$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If $f(x) = \{b^2 + (a-1)b + 2\}x - \int(\sin^2 x + \cos^4 x)dx$ be an increasing function of $x \in R$ and $b \in R$, then a can take value(s)

A. 0

B. 1
C. 2
D. 4

Question No. 6
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Let T be the triangle with vertices $(0, 0), (0, c^2)$ and (c, c^2) and let R be the region between $y = cx$ and $y = x^2$, where $c > 0$, then

A. $\text{Area}(R) = \frac{c^3}{6}$
B. $\text{Area}\ (R) = \frac{c^3}{3}$
C. $\lim_{c \to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = 3$
D. $\lim_{c \to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = \frac{3}{2}$

Question No. 7
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

The projection of line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ on the plane $3x + 2y + z = 0$ is/are

A. $\frac{x+1}{11} = \frac{y-1}{-9} = \frac{z-1}{-15}$
B. $\frac{x+12}{11} = \frac{y+8}{-9} = \frac{z+14}{15}$
C. $3x - 8y + 7z + 4 = 0 = 3x + 2y + z$
D. $3x - 8y + 7z + 4 = 0 = 3x + 2y + 2z$

Question No. 8
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Let matrix $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and A is 3 ordered square matrix such that $AB = BA$. If all entries of matrix A are whole numbers whose sum is 6, then-

A. If there are exactly 3 zero's in matrix A, then $\det(A) = 1$
B. If trace A = 6, then det(A) = 8
C. A is always an invertible matrix
D. there are 4 such matrices A.

Mathematics Numerical (Maximum Marks: 24)

Question No. 1
Numerical Type
The answer has to be filled into the input box provided below.

Let $z \in C$ satisfies the equation $\left|\frac{z+\bar{z}}{2\ \text{Re}(z)}\right| - |z| = \frac{2}{|\bar{z}|} - \left|\frac{z-\bar{z}}{\text{Im}(z)}\right|$. If locus of z is curve C_1 or $C_2 (C_1$ lies in $C_2)$ and chord AB of curve C_2 touches C_1 and from A and B two tangents are drawn to C_1 which meet at C lying on C_2 and if area of $\triangle ABC = \sqrt{k}$, then find the value of $[\frac{k}{4}]$.
[Note: $[y]$ denotes greatest integer less than or equal to y.]

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

If the coordinates of the point where the line $x - 2y + z - 1 = 0 = x + 2y - 2z - 5$ intersects the plane $x + y - 2z = 7$ is (α, β, γ) , then find the value of $(|\alpha| + |\beta| + |\gamma|)$.

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Number of solution(s) of equation
$x \in [0, \pi]$
$(\cos^5 x) + (\sin x)(\cos^4 x) - (\sin^4 x) + (\sin^3 x) + (\cos x)(\sin^2 x) - (\cos^2 x) = 0$ is

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

If $\cos^{-1}(2 - x^2) + \sin^{-1}(2 - x^2) + \tan^{-1} x \geq \frac{2\pi}{3}$ have no solution if $x < \alpha$ or $x > \beta($ for $\alpha < \beta)$, then $\{\frac{1}{\max(\alpha)} + (\min(\beta))^2\} =$ _____.

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

If $\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2$ then the value of a + b + c =

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

The number of ordered triplets (a, b, c) such that L.C.M. $(a, b) = 1000$, L.C.M. $(b, c) = 2000$ and L.C.M. $(c, a) = 2000$ is k, then $\frac{k}{10}$ is

Mathematics Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

Let $L_1 : 4x - 3y + 13 = 0$, $L_2 : 4x - 3y = 37$, $L_3 : 3x + 4y = 34$ are three lines in xy plane and $L_4 : (1 + \lambda)x + (1 - \lambda)y = 24$ is a variable line. P(a, b) is centre of circle which touches lines L_1, L_2 and L_3.

Question

On the basis of above information, answer the following questions :
Maximum value of $a + b$ is-

A. 13
B. 15
C. 17

D. greater than 17

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

Let $L_1 : 4x - 3y + 13 = 0,\ L_2 : 4x - 3y = 37,\ L_3 : 3x + 4y = 34$ are three lines in xy plane and $L_4 : (1 + \lambda)x + (1 - \lambda)y = 24$ is a variable line. $P(a, b)$ is centre of circle which touches lines $L_1,\ L_2$ and L_3.

Question

If $L_1,\ L_2,\ L_3$ and L_4 form a quadrilateral, then the value of λ for which slope of line L_4 takes least positive integral value is-

A. $\frac{4}{3}$
B. 2
C. $\frac{7}{5}$
D. $\frac{5}{4}$

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

If $(x + a)^n = \sum_{k=0}^{n} {}^nC_k x^{n-k} a^k$. Let $k({}^{10}C_k), b_k = (10 - k)({}^{10}C_k)$ and $A_k = \begin{bmatrix} a_k & 0 \\ 0 & b_k \end{bmatrix}$. If $A = \sum_{k=1}^{9} A_k = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$.

Question

Find the sum of digits of trace of the matrix A

A. 5
B. 14
C. 8
D. None of these

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

If $(x + a)^n = \sum_{k=0}^{n} {}^nC_k x^{n-k} a^k$. Let $k({}^{10}C_k), b_k = (10 - k)({}^{10}C_k)$ and $A_k = \begin{bmatrix} a_k & 0 \\ 0 & b_k \end{bmatrix}$. If $A = \sum_{k=1}^{9} A_k = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$.

Question

Which of the following is correct about 'ab'?

A. 'ab' has 4 prime factors
B. Largest prime factor of 'ab' is a three digit prime number
C. 'ab' is a 5 digit number
D. None of these